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Analysis of Transverse Muon Polarization in $K^+ \to \pi^0\mu^+\nu$ and $K^+ \to \mu^+\nu\gamma$ Decays with Tensor Interactions

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Abstract

The T violating transverse muon polarizations in $K^+ \to \pi^0\mu^+\nu$ and $K^+ \to \mu^+\nu\gamma$ decays due to tensor interactions are studied. The magnitudes of these polarizations over the allowed phase space are presented.

Although the standard model has been an enormous success in explaining experimental data, it is generally anticipated that there is new physics in higher energy regions. One of the long-shot efforts to exploring such new physics could be searches for CP violation (or T violation) outside of the Cabibbo-Kobayashi-Maskawa (CKM) paradigm [1].

It is well known that measuring a component of muon polarization normal to the decay plane in $K^+ \to \pi^0\mu^+\nu$ [2] or $K^+ \to \mu^+\nu\gamma$ [3] decays would signal T violation. These muon polarizations, called transverse muon polarizations ($P_\perp$), are related to the T-odd triple correlations:

$$\vec{s}_\mu \cdot (\vec{p}_\mu \times \vec{p}_\pi) \qquad \text{and} \qquad \vec{s}_\mu \cdot (\vec{p}_\mu \times \vec{p}_\gamma) \tag{1}$$

for the decays $K^+ \to \pi^0\mu^+\nu$ and $K^+ \to \mu^+\nu\gamma$, where $\vec{s}_\mu$ is the muon spin vector and $\vec{p}_i$ ($i = \mu$, π and γ) represent the momenta of the muon, pion and photon in the rest frame of K^+, respectively. It has been shown [4, 5] that, for $K^+ \to \pi^0\mu^+\nu$ decay, the transverse muon polarization defined by Eq. (1) is equal to zero in any theory of CP violation with the decay process via intermediate vector bosons (including the standard CKM model). It is also expected [6] that the CKM phase does not induce the muon polarization in $K^+ \to \mu^+\nu\gamma$ decay. Therefore measurements of these polarizations could be clear signatures of physics beyond the standard model. There are a number of different sources that might give rise to these polarizations, the most important ones being the weak CP violation from some kinds of non-standard CP violation models. The electromagnetic interaction among the final state particles can also make contributions, which are usually less interesting and could even hide the signals from the weak CP violation [7]. We shall refer to these final-state-interaction (FSI) contributions as theoretical backgrounds.

In this paper we will study the transverse muon polarizations in $K^+ \to \pi^0 \mu^+ \nu$ and $K^+ \to \mu^+ \nu \gamma$ decays in the precence of tensor interactions. We emphasize that these tensor interactions would undoubtedly be signals of new physics. Recently there has been considerable interest in the possibility of having tensor interactions in weak decays in connection with the experiments on $\pi \to e^- \bar{\nu} \gamma$ [8] and $K^+ \to \pi^0 e^+ \nu$ [9], in which tensor interactions have been introduced to explain the data. An analogous tensor interaction in $K^+ \to l^+ \nu \gamma$ $(l = e\ ,\ \mu)$ decays was discussed in Ref. [10]. For the decay $K^+ \to \pi^0 \mu^+ \nu$, we remark that the muon polarization induced by the tensor intreractions have been investigated previously, for example, in Refs. [4, 5]. Here we shall give a detail analysis on distributions of the transverse muon polarizations in terms of Dalitz plots and derive general constraints on the form factors. For the completeness, we will also include the scalar interactions in our discussions on $P_\perp(K^+ \to \pi^0 \mu^+ \nu)$.

We start by writing the decays as

$$K^+(p_K) \to \pi^0(p_\pi)\ \mu^+(p_\mu, s_\mu)\ \nu(p_\nu)\ , \tag{2}$$

and

$$K^+(p_K) \to \mu^+(p_\mu, s_\mu)\ \nu(p_\nu)\ \gamma(p_\gamma) \tag{3}$$

where p_i $(i = K\ ,\ \pi\ ,\ \mu\ ,\ \nu\ ,\ \gamma)$ are the four-momenta and s_μ is the four-spin muon polarization vector, respectively. For the decay $K^+ \to \pi^0 \mu^+ \nu$, we use the most general invariant amplitude adopted by the Particle Data Group [11]

$$\mathcal{M} \ = \ \mathcal{M}^{SM} + \mathcal{M}^{NSM} \tag{4}$$

where

$$\mathcal{M}^{SM} \ = \ \frac{G_F}{2} sin\theta_c [f_+(q^2)(p_k + p_\pi)^\alpha + f_-(q^2)(p_k - p_\pi)^\alpha] \bar{\mu} \gamma_\alpha (1 - \gamma_5) \nu \tag{5}$$

and

$$\mathcal{M}^{NSM} \;=\; G_F sin\theta_c[M_K f_S \bar{\mu}(1-\gamma_5)\nu + \frac{f_T}{M_K} p_K^\alpha p_\pi^\beta \bar{\mu}\sigma_{\alpha\beta}(1-\gamma_5)\nu] \tag{6}$$

are the amplitudes from the standard and non-standard interactions. Here f_S and f_T, known as the scalar and tensor form factors, are related to the effective scalar and tensor interactions, respectively and we have included the non-standard scalar part in the amplitude for the completeness. We note that in the case of $K^+ \to \pi^0 e^+ \nu$ decay, a fit of the experiment [9] gives that $|f_S/f_+| = 0.070 \pm 0.016$ and $|f_T/f_+| = 0.53 \begin{array}{c} +0.09 \\ -0.10 \end{array}$. These experimental values could also suggest large interactions of Eq. (6) in $K^+ \to \pi^0 \mu^+ \nu$ decay, which indeed have not been ruled out from the experimental data [11].

The probability of the decay as a function of the 4-momenta of the particles and the polarization 4-vector s_μ of the muon can be written as

$$dw = (1 + s_\mu \cdot P)\Phi' \rho/(2E_K) \tag{7}$$

where Φ' is a phase space factor,

$$\rho = \frac{d\vec{p}_\pi d\vec{p}_\mu d\vec{p}_\nu}{2E_\pi 2E_\mu 2E_\nu} \delta^4(p_K - p_\pi - p_\mu - p_\nu)(2\pi)^{-5}\,, \tag{8}$$

and P is the 4-vector muon polarization. In the kaon rest frame, the transverse component of P, i.e., transverse muon polarization, is given by

$$\vec{P}_\perp \;=\; \left(2Imf_S + \frac{1}{M_K^2}[m_\mu^2 - 2M_K(E_\mu - E_\nu)]Imf_T\right)\frac{\vec{p}_\pi \times \vec{p}_\mu}{\Phi} \tag{9}$$

with

$$\begin{aligned}\Phi \;=\; & (M_K - E_\pi - E_\mu)\left(2E_\mu - \frac{m_\mu^2}{M_K}\right) \\ & -\frac{1}{2}(M_K^2 + M_\pi^2 - m_\mu^2 - 2E_\pi M_K)\left(1 - \frac{m_\mu^2}{4M_K}\right)\,,\end{aligned} \tag{10}$$

where we have used $f_+ \simeq 1$ and $f_- \simeq 0$. Clearly, the non-zero contributions to $P_\perp \equiv |\vec{P}_\perp|$ in $K^+ \to \pi^0\mu^+\nu$ could arise if there are some non-standard effective scalar and/or tensor interactions.

At the present the best bound on the transverse muon polarization in Eq. (9) comes from the experiment at BNL [12] with the value of $(-3.1 \pm 5.3) \times 10^{-3}$, leading to the upper limit being about 10^{-2} at 90% C.L.. In the absence of the tensor form factor in Eq. (9), contours for the magnitude of the transverse muon polarization, $P_\perp$, are shown in Figure 1. Similar contours with assuming $f_S = 0$ are depicted in Figure 2. From $P_\perp^{expt}(K^+ \to \pi^0\mu^+\nu) < 10^{-2}$ and Figures 1 and 2, we find that

$$|Imf_S| \le 10^{-3} \qquad \text{and} \qquad |Imf_T| \le 10^{-3} \,. \tag{11}$$

Therefore, the transverse muon polarization in $K^+ \to \pi^0\mu^+\nu$ could be 10^{-2} without conflicting with the experimental constraints.

The muon polarization effects from these effective non-standard structures could be accessible to the underway experiment of E246 at KEK [13, 14], in which a sensitivity of 0.05% in $P_\perp(K^+ \to \pi^0\mu^+\nu)$ will be obtained, and future experiments in a kaon factory [15] as well. The well known examples which could give rise to the effective scalar interactions in Eq. (6) are the multi-Higgs and leptoquark models [5, 16, 17, 4], while the tensor ones could be induced from leptoquark models [5, 4].

Finally we remark that the theoretical backgrounds, i.e., *FSI* contributions, to the transverse muon polarization, can be ignored since they are expected to be $O(10^{-6})$ arising from two-loop diagrams [18].

We now study the decay of $K^+ \to \mu^+\nu\gamma$. In the framework of the standard model, the amplitude of the decay can be written as

$$\mathcal{M} \;=\; \mathcal{M}_{IB} + \mathcal{M}_{SD} \tag{12}$$

where $\mathcal{M}_{IB}$ and $\mathcal{M}_{SD}$ are the so called *inner bremsstrahlung* (IB) and *structure dependent* (SD) terms, given by

$$\mathcal{M}_{IB} = \frac{ieG_F}{\sqrt{2}} \sin\theta_c m_\mu f_K \epsilon^*_\alpha \bar{u}(p_\nu)(1-\gamma_5)\left(\frac{p^\alpha}{pq} - \frac{q\gamma^\alpha + 2p^\alpha_\mu}{2p_\mu q}\right) v(p_\mu) \tag{13}$$

and

$$\begin{aligned} \mathcal{M}_{SD} &= \frac{ieG_F}{\sqrt{2}} \sin\theta_c \epsilon^*_\alpha \left[pq\frac{F_A}{M_K}(\frac{p^\alpha q^\beta}{pq} - g^{\alpha\beta}) - i\epsilon_{\alpha\beta\rho\lambda}\frac{F_V}{M_K}p^\rho q^\lambda\right] \\ & \bar{u}(p_\nu)\gamma_\beta(1-\gamma_5)v(p_\mu)\,. \end{aligned} \tag{14}$$

Here ϵ_α is the photon polarization vector, f_K is the K decay constant, and $F_{A,V}$ are the axial, vector form factors defined by

$$\begin{aligned} < \gamma|\bar{u}\gamma_\alpha\gamma_5 s|K > &= -ie\frac{F_A}{M_K}\epsilon_\beta(pqg_{\alpha\beta} - p_\beta q_\alpha) \\ < \gamma|\bar{u}\gamma_\alpha s|K > &= e\frac{F_V}{M_K}\epsilon_{\alpha\beta\rho\lambda}\epsilon^\beta q^\rho p^\lambda\,, \end{aligned} \tag{15}$$

respectively. At the one-loop level in chiral perturbation theory, the form factors $F_{A,V}$ are found to be [19]

$$F_V = -0.0945\,, \quad F_A = -0.0425 \tag{16}$$

which agree with experiments. It is easily seen that the standard matrix elements in Eqs. (13) and (14) do not generate the T-odd triple correlation term in Eq. (1) for $K^+ \to \mu^+\nu\gamma$ decay because there is no relative phase between them at the tree level. It is clear that to have non-zero transverse muon polarization, a non-standard matrix element of new physics is needed. One possible candidate for such new physics in $K^+ \to \mu^+\nu\gamma$ is to have a tensor interaction given by

$$\mathcal{L}^T_{eff} = \frac{G_F}{\sqrt{2}} \sin\theta_c f'_T \bar{u}\sigma_{\alpha\beta}s\bar{\mu}\sigma^{\alpha\beta}(1-\gamma_5)\nu \tag{17}$$

where f_T' is a constant form factor. The possibility of having the interaction in Eq. (17) for $K^+ \to \mu^+\nu\gamma$ has been explored recently in Ref. [10] motivated by the experimental results [8]. With some arbitrary hypotheses [10] and results from the pion decays with PCAC approximation [20], the form factor f_T' is expected to be less than 6.2×10^{-2}. From Eq. (17), one obtains [10]

$$\mathcal{M}^T = \frac{ieG_F}{\sqrt{2}} \sin\theta_c \epsilon^{\alpha *} q^\beta F_T \bar{u}(p_\nu)\sigma_{\alpha\beta}(1+\gamma_5)v(p_\mu) \tag{18}$$

where the form factor F_T is defined by

$$< \gamma|\bar{u}\sigma_{\alpha\beta}\gamma_5 s|K >= -ie\frac{F_T}{f_T'}\frac{1}{2}(\epsilon_\alpha q_\beta - \epsilon_\beta q_\alpha) \tag{19}$$

with the form factor F_T being

$$|F_T| < 2.2 \times 10^{-2} \tag{20}$$

assumed in [10]. We note that the tensor interaction in Eq. (17) in principle could also lead to a contribution to $K^+ \to \pi^0\mu^+\nu$ decay [4]. However, the matrix element corresponding to Eq. (17) is suppressed by PCAC [10] unlike the relation in Eq. (19) for the case of $K^+ \to \mu^+\nu\gamma$ mode. We therefore believe that the decay of $K^+ \to \mu^+\nu\gamma$ is a more interesting one than $K^+ \to \pi^0\mu^+\nu$ to probe the tensor interaction of Eq. (17).

Similar to the discussions of $P_\perp(K^+ \to \pi^0\mu^+\nu)$, the transverse muon polarization for $K^+ \to \mu^+\nu\gamma$ arising from the interferences between the tensor term in Eq. (18) and the IB and SD terms in Eqs. (12)-(14) is found to have the following form

$$\vec{P}_\perp = \frac{4(1-\lambda)}{x}\left[(F_V - F_A)x^2 - \frac{2m_\mu^2 f_K}{M_K^3\lambda}\right] \cdot ImF_T \frac{\vec{p}_\mu \times \vec{p}_\gamma}{M_K^2} \cdot \frac{1}{\rho(x,y)} \tag{21}$$

with

$$x = \frac{2E_\gamma}{M_K}, \quad y = \frac{2E_\mu}{M_K},$$

$$\lambda = (x+y-1-r_\mu)/x\,,\ r_\mu = \frac{m_\mu^2}{m_K^2}\,, \tag{22}$$

and $\rho(x,y)$ is the normalized Dalitz plot density given by

$$\begin{aligned}\rho(x,y) &= \rho_{\rm IB}(x,y)+\rho_{\rm SD}(x,y)+\rho_{\rm IBSD}(x,y)\\ &\quad+\rho_{\rm T}(x,y)+\rho_{\rm IBT}(x,y)+\rho_{\rm SDT}(x,y)\end{aligned} \tag{23}$$

where

$$\begin{aligned}
\rho_{\rm IB}(x,y) &= 2r_\mu\left(\frac{f_K}{M_K}\right)^2 f_{\rm IB}(x,y)\\
\rho_{\rm SD}(x,y) &= \frac{1}{2}\left[(F_V+F_A)^2 f_{\rm SD^+}(x,y)+(F_V-F_A)^2 f_{\rm SD^-}(x,y)\right]\\
\rho_{\rm IBSD}(x,y) &= 2r_\mu\frac{f_K}{M_K}\left[(F_V+F_A)f_+(x,y)+(F_V-F_A)f_-(x,y)\right]\\
\rho_{\rm T}(x,y) &= 2|F_T|^2 f_{\rm TT}(x,y)\\
\rho_{\rm IBT}(x,y) &= 4\sqrt{r_\mu}\frac{f_K}{M_K}Re(F_T)f_{\rm IBT}(x,y)\\
\rho_{\rm SDT} &= 2\sqrt{r_\mu}Re(F_T)(F_V-F_A)f_{\rm SDT}(x,y)
\end{aligned} \tag{24}$$

with

$$\begin{aligned}
f_{\rm IB}(x,y) &= \frac{1-y+r_\mu}{\lambda x^3}\left(x^2+2(1-x)(1-r_\mu)-\frac{2r_\mu(1-r_\mu)}{\lambda}\right)\\
f_{\rm SD^+}(x,y) &= x\lambda[(\lambda x+r_\mu)(1-x)-r_\mu]\\
f_{\rm SD^-}(x,y) &= x(1-\lambda)[(1-x)(1-y)+r_\mu]\\
f_+(x,y) &= \frac{1-\lambda}{x\lambda}\left[(1-x)(1-x-y)+r_\mu\right]\\
f_-(x,y) &= \frac{(1-\lambda)x}{\lambda}-f_+(x,y)\\
f_{\rm TT}(x,y) &= \lambda x^2(1-\lambda)\\
f_{\rm IBT}(x,y) &= 1+r_\mu-\lambda-\frac{r_\mu}{\lambda}\\
f_{\rm SDT}(x,y) &= \lambda x^2(1-\lambda)\,.
\end{aligned} \tag{25}$$

In Figure 3, we show the contours for the magnitude of the transverse muon polarization in Eq. (21) induced by the tensor interaction. From the constraint in Eq. (20) we conclude that $P_\perp(K^+ \to \mu^+\nu\gamma)$ can be as large as 10%.

Although there is only one charged final state particle for the decay $K^+ \to \mu^+\nu\gamma$ like $K^+_{\mu 3}$ mode, the FSI due to electromagnatic interaction arises at one-loop diagrams because of the existence of the photon in the final state. Therefore it is expected [21] that the theoretical background, i.e., FSI, for the polarization in Eq. (21) is large, unlike the case in $K^+_{\mu 3}$ where the FSI is at the two-loop level. Recently, we have performed [22] a detail calculation on the FSI contribution to the polarization and found that $P_\perp^{FSI}(K^+ \to \mu^+\nu\gamma)$ is at the level of 10^{-3} in most regions of the decay allowed phase space.

Finally we note that the experiment E246 at KEK could also measure the muon polarization [14] in Eq. (21). Sensitivity at the level of 10^{-3} may be possible [23]. This will be a useful calibration for the experiment.

In summary, we have examined the T violating transverse muon polarizations in $K^+ \to \pi^0\mu^+\nu$ and $K^+ \to \mu^+\nu\gamma$ decays in the presence of non-standard interactions such as the tensor interactions. We have shown that the polarizations are expected to be large without conflicting with the current experimental data and they could be accessible at future experiments such as the underway experiment of E246 at KEK. Measuring the transverse muon polarizations in $K^+ \to \pi^0\mu^+\nu$ and $K^+ \to \mu^+\nu\gamma$ will be a clear indication of physics beyond the standard model and provide some insights into the origin of CP violation. In particular, these measurements could indicate the existence of the tensor interactions.

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Figure Captions

Figure 1: Dalitz plot of $P_\perp(K^+ \to \pi^0 \mu^+ \nu)$ for $Imf_T = 0$ with the values of contours: (a) - (e) being 0, 0.8, 1.4, 2.4 and 6.0 in the unit of $|Imf_S|$, respectively.

Figure 2: Dalitz plot of $P_\perp(K^+ \to \pi^0 \mu^+ \nu)$ for $Imf_S = 0$ with the values of contours: (a) - (f) being 0.2 0.1, 0, 0.3, 1.0, and 4.0 in the unit of $|Imf_T|$, respectively.

Figure 3: Dalitz plot of $P_\perp(K^+ \to \mu^+ \nu \gamma)$. The contours (a) - (d) are 0.5, 1.5 3.0 and 6.0 in the unit of $|ImF_T|$, respectively.

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